Lecture 6- "William Rowan Hamilton- A big time operator"

(1843 Trinity College Dublin Ireland)

1) Imagine living on the Argand Plane
   a) scalar point functions

2) Quaternions (dealing with Hamilton's imagination)
   a) Creating them - \((w + ai + bj + ck)\) The quaternion is the connection of the scalar point function value with its position vector.

   b) Multiplying a quaternion with a vector
      i) Invent some rules. <i| <j| <k| \(jk = i\) \(ki = j\) \(ij = k\) \(i^2 = j^2 = k^2 = -1\)
      ii) Multiply using binary multiplication (but add the new rules).
          \[(w + ai + bj + ck)(xi + yj + zk) = (wx + bz - cy) (i) + (wy + cx - az) (j) + (wz + ay - bx)(k)\]
      iii) Hamilton's representation for binomial product of quaternion and vector
           \[(w + ai + bj + ck)(xi + yj + zk) = (\text{dot product} + [x'i + y'j + z'k]) = (-\text{dot product} + \text{vector})\]
      iv) Hamilton product of two vectors.
           \[ (ai + bj + ck)(xi + yj + zk) = (bz - cy) i + (cx - az) j + (ay - bx) k\]
      v) Hamilton's view of the binomial multiplication of two vectors;
         you get a dot product and a vector product

3) Projectors
   If \(<x| = (1,0,0) <y| = (0,1,0) <z| = (0,0,1)\) and \(|c> = 1\)
   then \(<x|c> = 1 = c_1\) \(<y|c> = 2 = c_2\) \(<z|c> = 3 = c_3\)
   and \(|c> = |x><x|c> + |y><y|c> + |z><z|c>\)
   with \(|i><i|\) defined as a projector
   or phrased another way, \(|c> = \sum_{i}^{n}|i><i|\)
   a) Applications of Projectors for \(|c>\) and \(|j><j|c>\) as an example
      i) there is a vector \(|c'>\) such that \(|c'> = |c> - |j><j|c>\)
   or phrased another way, \(|c'> = \sum_{i}^{n}|i><i|\)
   ii) \(|s>\) = steady state vector, \(|c>\) current status vector, \(|t> = |c><c|s>\)
       if \((<t|t>)^{1/2}\) becomes smaller current status vector further from \(|s>\)

4) Rotations
   \[
   \begin{bmatrix}
   1 & 0 \\
   0 & 1
   \end{bmatrix}, \begin{bmatrix}
   S \\
   R
   \end{bmatrix} = \begin{bmatrix}
   2 & 0 \\
   0 & 2
   \end{bmatrix}, \begin{bmatrix}
   2 & 0 \\
   0 & 1
   \end{bmatrix}
   \]
   a) \(I |b> = |b>\) implies either no rotation or 360 degrees of rotation
   b) \(S |b> = |s>\) the vector \(|b>\) has been stretched but is in same direction
   c) \(R |b> = |r>\) the vector \(|b>\) has been rotated and (this time) also stretched.
   the first row element of column vector \(|r>\) is the dot product of matrix first row and the vector \(|b>\)