Lecture 5 - "How I came to love the Dot Product"

1) Tools for the talk

\[ (4,4) \begin{pmatrix} 2 \\ \frac{1}{8} \end{pmatrix} \begin{pmatrix} 0.24 \\ 0.96 \end{pmatrix} (0.71,0.71) \begin{pmatrix} 0.24 \\ 0.97 \end{pmatrix} \]

\[ 32^{1/2} = 5.66 \quad 68^{1/2} = 8.25 \]

2) Bra C Ket Notation \[ \langle b|C|k \rangle \]
   a) \[ \langle b| = a \] \(|1 \times n\) row vector \[ b\rangle \[ |k\rangle = a \] \(|n \times 1\) column vector

3) Scalar Product
   a) Operator symbol, operation, characterization
      i) shorthand notations ii) multiplication produces scalar iii) commutative
   b) Possible Dot Products
      i) with a single vector "self" dot product ii) with a different vector
   c) "Self" dot product
      i) "wird" vectors - vectors that are not normalized
         ii) "normalized vectors (vectors with a length of "unity")
      iii) how do you get normalized vectors?
         1) they come that way at "birth"
         2) they are sent through "normalization therapy"
   d) Vector "doted" with another vector
      \[ \cos \phi = \frac{\langle r|c \rangle}{\sqrt{\langle r|r \rangle \langle c|c \rangle}} \]
   e) Matrix elements (scalar products of vector sets [2 at a time]) \[ \langle r|, \langle s|, \text{and} \langle t| \]
      \[ \langle r|\rangle \quad \langle r|s \rangle \quad \langle r|t \rangle \]
      \[ \langle s|r \rangle \quad \langle s|s \rangle \quad \langle s|t \rangle \]
      \[ \langle t|r \rangle \quad \langle t|s \rangle \quad \langle t|t \rangle \]
   f) Graphic image of dot product
      i) projection vectors ii) directional cosines

4) Vector Product
   a) Operator symbol, operation, characterization
      i) \[ \langle b|x|k \rangle \]
      ii) multiplication produces vector iii) anti-commutative
   b) cross product vector-“parents” angle \[ \sin \phi = \frac{\langle d|d \rangle^{1/2}}{\langle b|b \rangle^{1/2} \langle c|c \rangle^{1/2}} \]
   c) Length of cross product vector \[ \langle d| \]
      \[ \langle d|d \rangle^{1/2} = \langle b|b \rangle^{1/2} \langle c|c \rangle^{1/2} \sin \phi \]
   d) \[ \langle d| \]
      \[ \langle d|d \rangle^{2} = \langle b|b \rangle \langle k|k \rangle - \langle k|k \rangle^{2} \]
   e) Components of \[ \langle d| \] when \[ \langle b| \text{and} |k\rangle \]
      are row and column vectors
      \[ \langle d| = (b_{2i} + b_{2j} + b_{2k}) \times (k_{1i} + k_{2j} + k_{3k}) \]
      or
      \[ \langle d| = b_{1} i + b_{2} j + b_{3} k \]
      \[ k_{1} k_{2} k_{3} \]

5) Triple Product
   a) Triple scalar product
      i) \[ \langle \langle b|x|c \rangle|e \rangle \]
      represents a volume ii) \[ D^2WR \]
      \[ \langle b|c|x|e \rangle \]
      doesn’t exist (you can cross the dot but not dot the cross)
   b) Triple vector product
      i) \[ \langle a|x|b\rangle \]
      \[ \text{or} \]
      \[ \text{AxBxC ("I'm back in the plane again")} \]
      ii) BAC-CAB rule \[ \langle b|a\rangle|c \rangle - \langle c|a\rangle|b \rangle \]
      = triple vector product answer